



University of Kragujevac
Faculty of Science

THE BOOK OF ABSTRACTS

The First Conference on Mathematics and Computer Science Teaching
August, 29–30, 2024, Kragujevac, Serbia



 **ТЕМАТСОМ**

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Faculty of Science



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Kragujevac, 2024

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ABOUT THE EVENT

The First Conference on Mathematics and Computer Science Teaching, TEMATCOM 2024, is a national conference with international participation. Teachers and assistants from the Faculty of Science, primarily the Department of Mathematics and Informatics, as an institution that has been educating mathematics and informatics teachers for over half a century, believed that this conference was the right moment to bring together all those involved in improving teaching practices, whether as researchers or practitioners. It will be an excellent opportunity for the exchange of scientific results, good practices, and the establishment of collaboration among colleagues from across Serbia, the region, and beyond.

The organizers' desire is to periodically organize the same conference, and the immediate reason for the first in the series is to celebrate the 70th birthday of [Prof. Branislav Popović](#), who has had a crucial impact on the reforms of study programs in mathematics (and informatics) in the field of methodology of teaching mathematics and related subjects focused on acquiring teaching competencies.

The official languages of the conference are Serbian and English.

The selected papers, following the reviewing process, will be published in the Conference Proceedings, which will also serve as the inaugural issue of the newly established journal of the Faculty, *Journal of Educational Studies in Mathematics and Computer Science (JESMAC)*.

The Conference Program includes plenary lectures, section lectures and posters organized in two sections:

- Research in the Teaching of Mathematics and Computer Science,
- School practice – yesterday, today, tomorrow.

Organization

The organizer of the conference is the Faculty of Science in Kragujevac, and the co-organizer is the Society of Mathematicians of Serbia – Branch Kragujevac.



The organization of the conference was financially supported by the Ministry of Science, Technological Development and Innovation.



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PLENARY LECTURES

How did TEMATCOM 2024 come about?

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This lecture aims to provide an insight into the events and opportunities that decisively influenced the TEMATCOM 2024 conference, shaped its content, aims and scope. A brief overview of the development of study programs for the education of mathematics teachers at the Faculty of Science in Kragujevac from its foundation until today will be presented. In fact, only the development of the part of study programs that leads to the realization of the teaching competencies of future mathematics teachers will be analyzed in detail. In addition to documenting the changes in the program, it will be emphasized that since the late 1990s these changes have been a part of a wider initiative that included the popularization of mathematics, the modernization of teaching and the strengthening of links between pre-university and university education, which was initiated by a group of mathematicians led by Prof. Dr. Branislav Popović. The foundations and development goals of the Institute of Mathematics and Informatics that were laid then left a deep mark and became the permanent mission of our institution. At that time, a special stimulating atmosphere was created for work in the field of teaching improvement, which we consider a privilege and a quality that should be permanently maintained. Full credit for that goes to prof. Dr. Branislav Popović, who, devoid of vanity, was always full of empathy for both colleagues and students, was able to ask the right questions and not impose answers. During the years that followed, the informal group he gathered expanded and developed, and its work produced many significant results for the teaching and learning of mathematics: scientific-research papers, professional papers, teacher training programs, textbooks, manuals for teachers, materials for the popularization of mathematics and the preparation of competitors. Research on various topics in the field of mathematics teaching methodology was initiated or encouraged by numerous, frequent and long conversations in the "Office for Beautiful Mathematics" (informal name of Prof. Dr. Branislav Popović's faculty office). The First Conference on the Teaching of Mathematics and Informatics - TEMATCOM 2024 is precisely the continuation of those discussions and was born from the desire of the members of the Institute of Mathematics and Informatics and the Mathematical Society of Serbia - Kragujevac Branch to expand the circle of those interested in improving the teaching of mathematics and to put the topics of teaching and learning mathematics (at all levels of education) in the focus of the professional public. The conference also represents a kind of invitation to all interested colleagues for cooperation and discussion on these issues.

Key words: *mathematics teaching methodology, research on teaching and learning mathematics*

Three Big Questions that Shape Teaching

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What? To whom? How? The answers to these questions significantly determine all segments of education, from the general principles on which an educational system is based to the preparation of individual teaching units. It has long been recognized that there are no "timeless" answers and that they, as a rule, significantly depend on the general value system of current socio-cultural circumstances. The lecture will attempt to find answers appropriate to today's circumstances and trends in education, aligned with the most important goals of learning mathematics. We will seek answers somewhere between the well-known ideas of great modern mathematics educators, which have yet to be implemented in practice, and classroom examples that have yet to become guidelines for serious methodological analysis. The answers will be summarized as didactic principles on which any form of teaching organization should be based: 1) the principle of conciseness; 2) the principle of adaptation; 3) the principle of activity. The general discussion will be supported by various illustrations composed in the form of tasks about Platonic solids. The tasks will be selected to highlight the place that this ancient mathematical topic deserves in modern mathematics teaching at all levels of education.

Key words: *basic didactic principles, Platonic solids*

Generative Artificial Intelligence as Support in Education: Opportunities, Challenges, and Examples

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In recent years Generative Artificial Intelligence (AI) and its applications have been one of the most popular topics both in the technological world and among technology users. The rapid advancement of large language models, such as GPT, in generating content across different modalities (text, images, sound, etc.) has made them indispensable in everyday life and business. Generative AI is particularly relevant in modern education, both in terms of developing educational technologies and enhancing the teaching process, as well as preparing students for responsible and purposeful use of AI.

When it comes to modern educational trends, teachers are expected to enable active learning and have an individualized approach to each student. Therefore, teaching models such as problem-based learning, project-based learning, and game-based learning are becoming the norm. It is noticeable that there is a shift towards moving the teacher's role from lecturer to mentor, which implies a different engagement in the preparation and implementation of teaching, as well as in monitoring students. Whether and to what extent this is expected to take root largely depends on the resources (time, material, and technological) available to the teacher. Generative AI has the potential to be a key factor in this change. The topic of this lecture is the consideration of the possibilities of using generative AI in the teaching process in both primary and secondary schools. It aims to introduce teachers to generative AI technologies by presenting how they work and analyzing the possibilities, limitations, and risks of working with AI. The lecture will provide practical examples of using AI tools in preparing teaching materials.

Key words: generative artificial intelligence, large language models, instructional strategies

SECTION 1: RESEARCH IN
THE TEACHING OF
MATHEMATICS AND
COMPUTER SCIENCE

About a class of irrational equations

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Equation $\sqrt{x^2 - 7} = x + 7$ is the most common form of irrational equations encountered in mathematics education in high schools. Assuming we need to solve an irrational equation in the set of real numbers where the square root on the left side of the equation has been "moved" to the right side. At first glance, it seems there are no major issues since, under classical conditions $x^2 - 7 \geq 0$ and $x + 7 \geq 0$, by squaring the given irrational equation is transformed into the algebraic equation $(x^2 - 7)^2 = x + 7$, or equivalently $x^4 - 14x^2 + 49 = x + 7$. There are many methods for solving this resulting algebraic equation, but the question arises: what if the problem is more complex, such as the equation $x^2 - 77 = \sqrt{x + 77}$? It's clear that the corresponding algebraic equation can apply the theorem on rational roots of polynomials, but now it's much more challenging because the constant term of the polynomial $77^2 - 77 = 77 \cdot 76$ has significantly more divisors, and the question is whether the obtained polynomial even has rational roots. The goal of this paper is to present a non-standard approach to solving irrational equations of the form $x^2 - a = \sqrt{x + a}$, where a is a real number, and to raise a series of questions regarding the existence and nature of solutions to the considered equation.

Key words: irrational equations, algebraic equations

How had I begun to understand combinatorics?

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The path to acquiring mathematical knowledge is as beautiful and unique as the concept itself. This particularly applies to the development of combinatorial thinking, whose beginnings can be traced back to early childhood. The author firmly believes that our achievements in professional work are deeply correlated with our level of understanding of combinatorics. This presentation offers a brief personal retrospect on my initial discoveries and the life environment during my education in the Mathematical Workshop for Youngsters in Kragujevac, with a special focus on the sum rule learned from Prof. Branislav Popović, a celebrated conference attendee, and the double counting mastery achieved only when I became a mathematics teacher. The ability to generalize mathematical assertions often proves to be a natural progression built upon early and well-established foundations.

Key words: *combinatorics, addition rule, generalization, double counting*

Mathematical modeling in the study of definite integrals

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Studying definite integrals in high school is of great importance for further education in mathematics and technical sciences. The significance of studying definite integrals lies in their applications in mathematical modeling. Therefore, the application of mathematical modeling is expected to contribute to better learning outcomes in understanding definite integrals. The paper will present a study conducted with high school seniors from the "Svetozar Marković" Gymnasium in Novi Sad. A problem-based approach was applied, where students used dynamic software to determine volumes of various solid bodies from their environment. The approaches students used in problem-solving, cognitive conflicts they encountered, and methods for overcoming these conflicts will be analyzed.

Key words: *dynamic software, cognitive conflicts, solid bodies, definite integral*

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Teacher and teaching mathematics, developing (mathematical) thinking

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Each period of social development is marked by a corresponding pedagogy and accompanying didactics, that is, special didactics. In that sense, the methodology of teaching mathematics is also determined by various influences induced by complex relations between different factors in the educational field. Teachers (within the area of teaching mathematics) are performers in a demanding endeavour of developing students' mathematical thinking. The assumptions for high-quality education work in the segment of teaching and learning mathematics are contained in the initial qualifications, reception of didactic reality, ability/training for permanent improvement in practice as well as the existence of various/adequate forms of the support service for teachers. This text aims to direct attention towards the mentioned key elements of hallmarks of the teaching mathematics vocation in our times and situates this vocation within the reality of the educational process, together with the peculiarities of teaching mathematics. Recognizing this special character does represent a challenge for teachers which can be resolved by connecting it with didactics related to other subjects in the common educational work.

Key words: learning, mathematics teacher, teaching mathematics, developing thinking, developing mathematical thinking

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Balanced ScoreCard Model in the context of teaching mathematic

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Educational work and particularly teaching mathematics require monitoring effectiveness and efficiency, that is, successfulness of the learning process. The model of the Balanced ScoreCard (BSC), originally and dominantly designed to monitor performance of business subjects, gives good results in application and advancement in practice of different entities in the public sector as well as among non-profit organizations. Research into application of BSC, spanning many years and even decades, has transformed this model into a universal approach for describing, monitoring, measuring and, based on the data, making decisions and correcting activities that are intended to provide superior results. This paper shows the manner in which it is possible for teaching and learning mathematics to be grasped by the perspectives of the BSC, that is, to demonstrate the possibilities of efficient application of the BSC approach in designing and realization of teaching mathematics.

Key words: *BSC, Balanced ScoreCard, teaching mathematics, educational process*

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Study on the Intuitive Understanding of Probability among Primary School Students: A Case Study from Serbia and the Czech Republic

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This study emphasizes the importance of teaching probability in primary school, with a focus on the need to integrate intuitive reasoning about randomness with structured school learning. Using the examples of Serbia and the Czech Republic, where primary school students do not learn about probability as a part of the compulsory mathematics curriculum, the study tested intuitive understandings of basic probability concepts. The test administered to students included problems of varying complexity and difficulty related to tossing one or more coins or dice. Analyzing the responses of 600 students aged 12-15 from both countries, the study aimed to uncover both well-founded understandings of randomness among students and prevalent misconceptions about probability. Considering the research results, educational institutions can reassess and update their curricula to better support students' development in this important mathematical concept. Integrating basic elements of probability theory into the primary school mathematics curriculum could significantly enhance students' understanding of random phenomena and probability laws. Such an approach would not only improve students' competency in mathematics but also foster the development of critical thinking and problem-solving skills, and the application of mathematics in real-life situations.

Key words: students' intuition, probability, teaching and learning programs, students' misconceptions

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STREAM Approach in Education: Models and Implications for Practice

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STREAM (Science, Technology, Reading, Engineering, Arts, Mathematics) approach in education is a transdisciplinary approach that integrates two or more disciplines: natural sciences, technology, reading, engineering, arts, and mathematics, with a special emphasis on applying learned concepts to solve real-world problems. This paper aims to present the theoretical significance of the STREAM approach in education and offer STREAM models and guidelines for successful implementation in classroom teaching. The paper will extensively describe the Symmetry Processing model using the STREAM approach, utilizing dynamic mathematical software GeoGebra, and integrating mathematics with other disciplines. Additionally, guidelines will be provided on how other mathematical content can be processed using the STREAM approach. These models are created based on perceptions of teachers regarding their experiences in applying the STREAM approach. The created models serve teachers in acquiring new knowledge and enhancing existing knowledge about the application of the STREAM approach. They also provide opportunities for implementing the STREAM approach within the existing educational system in Serbia.

Key words: STREAM, integration, STREAM models, GeoGebra

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The impact of intelligent tutoring systems and artificial intelligence on students' motivation and achievement in STEM education: a systematic literature review

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In recent years, STEM education, which includes science, technology, engineering, and mathematics, has seen significant and dynamic progress. These advancements are marked by the proliferation of scientific understanding and the emergence of cutting-edge educational resources that rely on artificial intelligence (AI) technology, among other factors.

A promising avenue for advancing formal education is presented by intelligent tutoring systems (ITSs), which provide intelligent instruction and feedback, thus facilitating a more personalized and practical learning experience. This study explored emerging trends and the feasibility of integrating intelligent tutoring systems in STEM education. A systematic literature review was carried out following PRISMA guidelines. A total of 24 studies were included, selected based on predefined inclusion criteria related to the research objective. The analysis indicated an increasing interest in intelligent tutoring systems within STEM education between 2019 and 2024.

Moreover, most of the research conducted so far has been focused on the K-12 education system and higher education institutions. The research began by examining the impact of Intelligent Tutoring Systems on enhancing student motivation and overall academic performance in STEM education courses. The results substantiate the assertion that integrating artificial intelligence into intelligent tutoring systems has a beneficial effect on student motivation and achievement in STEM education. Additionally, students' existing knowledge of a STEM subject enhances their engagement and motivation to learn when using intelligent tutoring systems. Integrating intelligent tutoring systems into STEM education has enhanced student motivation and academic achievement.

Key words: Intelligent Tutoring System, Artificial Intelligence, STEM education, Students' Motivation, Systematic Literature Review

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The role of educators in developing IT skills in preschool children

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The development of IT skills at an early age is a crucial component of education. This paper explores the role of preschool teachers in developing IT skills in preschool children, with a particular focus on the use of digital didactic games. Teachers are responsible for creating a stimulating and safe environment that allows children to explore technology through play and interactive activities. Through qualitative research, including interviews and observation of activities in kindergartens, key approaches and challenges in the work of teachers, as well as examples of good practices, were identified. The results show that teachers play a key role in fostering an interest in information technology among children. Through the use of digital didactic games, children acquire basic computer knowledge and develop logical thinking and problem-solving skills. These games are particularly effective because they combine fun and learning, maintaining high motivation among children. Teachers report that the biggest challenges in their work are the lack of adequate resources and technological support, as well as limited time to conduct these activities. Recommendations for practice include integrating IT activities into the regular curriculum, providing adequate technological resources, and continuous training for teachers. It is concluded that with proper support and resources, teachers can significantly contribute to the development of IT skills in preschool children.

Key words: IT skills, preschool children, digital didactic games

Do We Have an Average Mathematics Student?

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The structure of a primary school classroom in the Republic of Serbia is almost always heterogeneous. This poses a significant challenge for mathematics education. If teaching is largely adapted to the average student, the question arises as to how we define or recognize such a student. In this paper, we will analyze the grading in mathematics education, which significantly deviates from expectations. Using data on the grades of eighth-grade students from 2013 to 2019 and their final grades in mathematics, it was found that the grade distribution follows a "U" shape, meaning that the smallest percentage of students have grades 3 and 4. To gain an initial insight and create conditions for more detailed research into the causes of such a grade distribution, a survey of teachers' attitudes and opinions on this phenomenon was conducted through a focus group. Teachers highlighted strong internal and external motivation of students, primarily driven by parents, as one of the reasons for this grade distribution. The results of the analysis of teachers' attitudes and opinions have raised many questions that point to further and more detailed research into grading methods. Additionally, this paper will refer to the potential problems of such a grade distribution and the consequences that can already be observed today.

Key words: mathematics grade, evaluation, importance of grading, grade distribution

The impact of peer assessment on student achievement in the web programming course

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It is well known that peer assessment offers numerous benefits, including improving work quality and developing critical thinking skills. Over two years, in a web programming course, research was conducted aimed at examining the impact of feedback and peer assessment on student achievement in this subject. The web programming course demands an integration of diverse knowledge and skills, such as comprehension of the development environment, mastery of various tools, and familiarity with programming languages and frameworks used in web page creation. Besides technical knowledge, attention to the needs of the potential client for whom the website is being created is also expected.

In the mentioned course, students were tasked with developing a website on a given topic according to specific criteria. The research results showed that students who participated in peer assessment, through analyzing their peers' work, providing qualitative assessments, reflecting on their own work based on received comments, and making necessary corrections, created content that surpassed the quality of the products (websites) created by their peers who did not participate in this process. This approach enabled students to develop necessary competencies in assertive communication through peer assessment and feedback, making them more successful in creating web content and meeting various requirements and expectations.

Key words: *Peer Assessment, Web Programming, Higher Education*

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Interactive textbook on algorithms and data structures

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Algorithms and data structures are an essential part of computer science studies. In this presentation, a digital, interactive version of the textbook for the course "Construction and Analysis of Algorithms" at the Faculty of Mathematics, University of Belgrade, will be showcased. The course material includes advanced data structures, graphs, algebraic algorithms, text processing algorithms, and geometric algorithms. What sets this textbook apart from traditional textbooks is the multitude of interactive components implemented in JavaScript, using modern web technologies, which enables students to better understand the functioning of the algorithms and data structures described. In addition to following the step-by-step execution of an algorithm, the textbook often requires readers to independently perform certain procedures, verifying the correctness of the results. This provides students with feedback on how well they understand the process. This approach aligns with the constructivist teaching method, where students do not passively acquire knowledge but must take an active role throughout the learning process. The textbook is freely available and can be used by all computer science students studying in the Serbian language. The libraries developed for the implementation of this textbook are also freely available and can be used for the development of similar textbooks in other areas of computer science or mathematics.

Key words: interactive digital publications, algorithms, data structures

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Digital Platforms for Final Exam Preparation in the Educational System of Serbia

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In this paper, digital platforms for final exam preparation in the educational system of Serbia are analyzed. Two key platforms developed by the Working Group for Educational Software of the Faculty of Mathematics at the University of Belgrade are considered: eVežbaonica and Final Exam, which are designed to help students acquire and verify knowledge in various subjects. The Final Exam platform offers tasks in mathematics and the Serbian language, grouped by topics and levels, allowing students to test their knowledge through independent practice tests and exercises from different areas. The platform also provides customized tasks for students included in inclusive education, enabling them to enhance their knowledge through digital technologies, thus promoting independence and continuity in their work. The eVežbaonica platform, created in collaboration with the Ministry of Education, the Institute for Evaluation of Quality of Education and Upbringing, eGovernment, the Faculty of Mathematics, and the Faculty of Organizational Sciences at the University of Belgrade, offers over 4000 tasks in seven subjects (mother tongue, mathematics, physics, chemistry, biology, geography, and history) for final exam preparation. This platform features a specialized markup language that allows easy entry of different types of tasks (multiple choice, short answer, table selection, etc.) and does not restrict authors from using predefined templates when designing tasks. Both platforms are developed using modern web technologies and are adapted to different devices, including computers, tablets, and smartphones. Both platforms represent significant progress in the digitalization of education in Serbia, providing free resources for students and supporting diverse learning styles and needs in preparing for the final exam.

Keywords: *final exam, eVežbaonica, educational platforms*

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Encouraging Research Orientation in the Education of Students – Future Mathematics Teachers

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In this paper, we address the issue of developing a research orientation among third-year students at the Faculty of Science, University of Kragujevac, in the Mathematics Teacher Module. We will provide a detailed account of the process and results of a project that emerged from collaboration among professors from different scientific disciplines at the Faculty of Science in Kragujevac. Within this project, the cooperation between this higher education institution and the Kragujevac School Administration of the Ministry of Education of the Republic of Serbia and the First Grammar School of Kragujevac was intensified. The project was implemented during the summer semester of the 2022/23 academic year, involving a sample of 12 students. These students conducted mini-research projects on topics covered in the School Pedagogy course during their internships at the First Grammar School of Kragujevac as part of the Professional Practice 2 course. The students shared their findings, both in written and oral form, with their colleagues and course teachers at the Faculty of Science. Selected topics were subsequently presented to several teachers (mostly mathematics teachers) employed in the Šumadija District.

Key words: *mathematics students, future teachers, research, collaboration*

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Incompletely Formulated Problems – Pros and Cons

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Problem-solving holds a central place in mathematics education. In contemporary school practice, the requirement to contextualize mathematical knowledge urges students to become familiar with incompletely defined problems. On the other hand, there is logical, terminological, and professional reluctance toward poorly formulated tasks, and sometimes incompletely formulated problems are incorrectly or rightly identified as poorly formulated. This paper aims to explore the role of incompletely formulated tasks in mathematics education. A theoretical epistemological partial structural analysis of examples was applied to investigate the methodological potential of such problems in teaching, including consideration of the positions of participants in problem-solving, both teachers and students, using the three classifications of problems by Yea, Foster, and Maker-Shiver within the framework of the "pedagogical perspective". It was concluded that a programmatic orientation towards realistic mathematical education provides theoretical support for a more diverse selection of mathematical tasks in the school curriculum, including incompletely formulated problems. However, the impact of incorporating such problems into school practice requires validation through future research.

Key words: *mathematical problem, problem formulation, problem space, classification*

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What do the solutions to an open-ended problem reveal about students' knowledge?

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Solving mathematical problems provides insights into students' procedural and conceptual knowledge. In addition to having multiple solutions, open-ended problems can often be solved in various ways, allowing for deeper insights into students' competencies. This study investigates the didactic potential of a multi-solution open-ended problem in the domain of fractions. The achievements of 5th-grade elementary school students were analyzed to determine problem-solving success, examine solution strategies, identify types of errors, and consequently identify what prevents students from successfully solving the problem in its entirety. The problem analyzed here is a part of an instrument developed to study how students solve open-ended problems. The empirical research included 121 eleven-year-olds from various urban and suburban schools in Croatia. The analysis of results shows that students used a range of strategies, many of which did not lead to correct solutions, and that students demonstrated limited conceptual knowledge about fractions, which eventually hindered or prevented them from successfully discovering all correct solutions. Assuming the limited experience of students and their awareness that problems can have more than one correct solution, the methodological implications of the study's results were highlighted, particularly the need for teachers to become familiar with the potential of these problems and to be trained to use them diagnostically in teaching practice.

Key words: *open-ended problem, fractions, solving methods, errors*

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Investigating the Impact of Technological Pedagogical Content Knowledge (TPACK) on the Adoption of E-Learning in STEM Education

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Examining the development of Technological Pedagogical Content Knowledge (TPACK) in STEM education through e-learning and assessing the acceptance of e-learning are crucial for understanding how TPACK can enhance the effectiveness of teaching STEM subjects in a digital learning environment. The aim of this research is to investigate the variables that influence TPACK and the acceptance of e-learning in order to improve pedagogical innovations in STEM education. The variables studied include the intention to use technology in STEM education (BISTEM), subjective norm, perceived usefulness of technology, perceived ease of use of technology (PEU), quality of teachers in e-learning (EL_TK), usefulness of online group work (EL_PU), facilitating conditions in e-learning, technological knowledge, content knowledge (CK), and TPACK. The examination was conducted by surveying 186 prospective teachers who participated in online group work, followed by the application of multiple linear regression on the collected data. The results showed that TPACK is predominantly influenced by EL_PU, CK, and PEU. On the other hand, the intention of future teachers to use technology in STEM education is influenced by TPACK, CK, and EL_TK. The variance explained by the predictors in the equations amounts to 48.2% in TPACK and 52.8% in BISTEM. The results highlight the importance of primarily improving the perceived usefulness of online group work, content knowledge, and ease of use of technology for the development of TPACK in STEM education through e-learning. Additionally, fostering the development of TPACK, especially focusing on content knowledge, and ensuring high-quality teacher interaction in e-learning are emphasized as key factors for educators who aim to develop the intention of future teachers to implement technology in STEM education.

Key words: TPACK, STEM education, e-learning acceptance, pedagogical innovations, future teachers

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Cognitive conflicts of students during the computation of definite integrals

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Tall and Vinner (1981) explored students' approaches to solving posed problems in mathematics education. Their models of concept image and concept definition enable the analysis of students' representations of mathematical concepts. Using concept image and concept definition, Tall and Vinner (1981) distinguish aspects of mathematical knowledge that are, on one hand, given by formal definitions, and on the other hand, based on students' subjective constructions. In this paper, we will present a study conducted to examine how students understand and compute definite integrals of variously defined functions. Special attention will be given to overcoming cognitive conflicts that arise among students when working with integrals, such as functions given piecewise. The sample consisted of 20 gifted final-year students from "Jovan Jovanović Zmaj" Gymnasium in Novi Sad, who completed a written task involving definite integrals and their applications. The students' results were compared with findings from research conducted by Rasslan and Tall (2002). Analysis of the students' results led us to conclude that cognitive conflicts were most pronounced among students when adopting the definition of the improper integral, viewed as a special case of the definite integral. Students also encountered difficulties in computing definite integrals of differently defined functions and applying definite integrals to calculate the area defined by function that changes sign within a given domain.

Key words: definition of concept, cognitive conflicts, definite integral of a function, concept visualization

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Introduction to Machine Learning through Examples from Art

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This paper presents projects that can be used in secondary school education to introduce students to the field of machine learning. The proposed projects include examples from art, making them visually engaging and potentially interesting for students to explore independently and deepen their knowledge further. The main idea is to explain key concepts of artificial intelligence and then apply that knowledge in implementing these projects. Artworks and photographs used in the projects correlate with visual arts, geography, history, biology, and other subjects. The libraries used in project implementation provide students with a real insight into problem-solving methods using machine learning techniques. The proposed implementation methods assume familiarity with the Python programming language. The suggested project delivery allows teachers to integrate them into their teaching organization, creativity, and teaching style. Overall, this work facilitates the integration of machine learning into the school curriculum and encourages students to explore the possibilities of artificial intelligence through art in an interactive and enjoyable manner.

Key words: *artificial intelligence, machine learning, art, neural networks*

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The Application of New Technologies in Computer Science Teaching

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In contemporary society, rapidly evolving under the influence of new technologies, it is crucial to recognize the importance of applying these technologies in computer science education. Changes in society and the workplace urge students to develop various skills and competencies supported by technological tools. This imposes challenges on the education sector in its effort to adapt its methods and resources to enable students to acquire necessary knowledge and skills. The aim of this research is to examine the implementation of new technologies in computer science education and explore their effects on the learning process. Special emphasis is placed on analyzing the impact of technology on student outcomes and motivation for learning. The study employed a mixed-methodological approach, including quantitative and qualitative methods, surveying 150 participants. Participants included computer science teachers and students from various educational institutions. The research results indicated that the implementation of new technologies in computer science education can have a significant impact on student learning and motivation. Students who had access to new technologies showed better results on tests and higher levels of engagement in the learning process. Additionally, these students expressed greater interest in the subject and a higher readiness to participate in learning activities. Based on these findings, the application of new technologies in computer science education was concluded to be essential in improving the educational process. Implications for contemporary educational practices include the need for teacher training to successfully integrate technology into teaching. Furthermore, it is important to promote a culture of innovation and exchange of best practices among educational institutions.

Key words: educational resources, new technologies, digital literacy, innovations in teaching, computer science education

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Application of Augmented Reality in Solving Stereometry Problems

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Solving problems in the field of stereometry has always been a challenge for both teachers and students. The most common difficulties that arise when solving problems in this area relate to the representation and visualization of a 3D environment. Today's computer technologies have developed specific tools, such as applications that support work in a 3D environment, and especially work in so-called augmented reality (AR), which allows the representation of geometric bodies in a real environment. This paper will present the 3D Calculator application of GeoGebra software and its specific module for working in augmented reality. Several selected stereometry problems will be shown and explained in detail, demonstrating how to solve them using the augmented reality application.

Key words: *GeoGebra, 3D Calculator, augmented reality, stereometry*

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Functional Teaching of Mathematics

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Effective modern mathematics teaching represents a symbiosis of traditional approaches and contemporary methods, without which functionality and achievement of set goals cannot be expected. This paper provides answers to some important questions: how a teacher should prepare for mathematics classes, what materials can be used, how to design assessments, how to select tasks considering standards and students' prior knowledge. It will also showcase a selection of tasks that promote functional thinking and track outcomes, demonstrating how standards aid task creation. Besides the importance of selecting appropriate tasks for mathematics classes and paying attention to their solutions, tests oriented towards outcomes and functional knowledge are crucial for the functionality of mathematics teaching. The paper will illustrate how to create effective tests or written assignments that clearly communicate to students, parents, and teachers and that show what the student knows, can do, and understands in a specific area after completing the test and receiving a certain grade.

Key words: *functional knowledge and assessments, achievement standards, outcomes*

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Learning mathematics using different softwares

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We will present a study conducted with first-year students of the Subotica Tech College of Applied Sciences during the 2023/24 academic year as a part of the subject Mathematics 2. The aim of the research was to examine the impact of different softwares on the efficiency of learning mathematical analysis. Special attention was given to the qualitative analysis of student assignments from exercises and written exams. During the exercises, students worked in small groups and used a software of their own choice, while they completed exam tasks individually without the use of software.

Key words: *software, functions, collaborative learning, mathematical analysis, definite integral*

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Reasoning of Artificial Intelligence in Solving Logical Tasks

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One of the tasks of mathematics education is to develop students' logical thinking. Logical tasks that require a certain level of sharpness, ingenuity, and rational reasoning present a good challenge that can further motivate students to engage with mathematics. The progress of science and technology has led to the emergence of numerous tools that can be used in mathematics education. With the advent of artificial intelligence, these tools have taken on a more active role, as communication with users reaches a new "user-friendly" dimension resembling human-to-human interaction. The unpredictability of the flow and the speed of development of artificial intelligence raises the question of whether it will ever reach or surpass the level of human sharpness and ingenuity. This paper illustrates the approximate and partially accurate reasoning of artificial intelligence in solving logical problems.

Key words: logical tasks, artificial intelligence

Mathematical Models in Medical Sciences and Their Integration into Mathematics Education

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Mathematical models are fundamental tools for understanding complex biological and medical phenomena as they enable prediction, diagnosis, analysis, and optimization. This paper explores the application of mathematical models within medical sciences and the possibilities for their use in enhancing mathematics education. The paper presents several significant models, such as disease spread models, tumor growth and development models, pharmacokinetic, hemodynamic, and genetic models, through a detailed analysis of the mathematical models used to describe real-world phenomena being modeled. A particular emphasis is placed on the possibilities of integrating these models into high school mathematics education, specifically on how complex mathematical content such as functions, differential and integral calculus can be made more accessible to students through real-world examples. The paper discusses the advantages and challenges of integrating these models into the mathematics curriculum, emphasizing the difficulties teachers may face and the impact on student motivation and achievement. The results of the paper indicate that mathematical models from medical sciences can be successfully integrated into mathematics education, enhancing the understanding of complex mathematical content, boosting student motivation and achievement, and increasing student interest in STEM disciplines.

Keywords: *mathematical models, medicine, mathematics education*

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Improving the understanding of mathematical proofs with the help of computer proof assistants

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Understanding and writing proofs of mathematical theorems is a key aspect of learning mathematics and developing the logical apparatus that enables success in this field. The importance of proof is also reflected in the facts that, on the one hand, on exams students are expected to derive a proof, and on the other hand, mathematical theorems are most easily comprehended through the explanation of its proofs.

The observations presented in this paper are based on the experience of working with students in the elective course "Introduction to interactive theorem proving" held in the 4th year of undergraduate studies at the Faculty of Mathematics in Belgrade. When students first encounter interactive theorem proving, understanding the notion of the proof seems to be at a relatively low level. The course is implemented using the Isabelle theorem prover and consists of two parts. The first part of the course is proving elementary mathematical theorems (about logic, sets, functions, relations, natural and real numbers) by natural deduction rules, case analysis, and induction. The second part of the course is verification of functional programs. At the end of the course, students master the knowledge to such an extent that they are able to successfully formally prove tasks and solutions from international mathematical Olympiads or to verify certain algorithms.

Our recommendation would be to consider introducing a course of this type potentially at other faculties as well, and at earlier years of undergraduate studies. It could potentially be worth considering using interactive theorem provers as a part of basic mathematical logic courses. The community that uses formal proving in Serbia is very small, and the expansion of that community would certainly contribute to the development of critical thinking, logic, and long-term knowledge acquisition.

Key words: proof understanding, formal proof, interactive theorem proving, Isabelle, teaching mathematics

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Unified multiplication and division table – a contemporary approach

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The paper presents a contemporary approach to integrating the multiplication and division tables, thereby making contribution to the field of mathematical education. Traditional methods of teaching multiplication and division operations are often fragmented, leading to misunderstandings and difficulties in applying acquired knowledge. To overcome these challenges, a unified table is proposed that integrates both mathematical processes, enabling students to develop a deeper understanding of the relationships between multiplication and division. Additionally, from the same table, square roots and squares of numbers can be determined, thereby expanding its usability and functionality. This table can be applied from the second grade of primary school, providing students with earlier access to more complex mathematical concepts. The paper discusses the methods and approaches used in creating this table, as well as its interpretation and practical application in teaching. Through theoretical analysis aligned with the NTC learning system, it is indicated that this innovative approach significantly contributes to improving mathematical literacy and enhancing students' competencies in solving mathematical problems.

Key words: multiplication table, division table, square root of a number, square of a number, and teaching methodology in mathematics

The area of a sphere and the volume of a ball – some interesting phenomena

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The surface of a sphere and the volume of a ball are topics in the scope of the curriculum in both primary and secondary schools. When studying these topics at school, the appropriate formulas are often only stated without a proof or at least explanation. In this talk, we will show various ways to derive the formulas for the surface area of a sphere and the volume of a ball if their radii are given. We will show how these formulas are derived using integral calculus, but also in an elementary way, i.e. without using integrals. In particular, we will consider the areas of the parts of a sphere and the volumes of the parts of a ball (spherical cap, sector, segment, shell and wedge). Finally, we will show some interesting phenomena related to these notions and formulas.

Key words: *The surface of a sphere, the volume of a ball*

Transition from elementary to advanced mathematical thinking

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Based on Piaget's developmental theory, Skemp, Tall, and other authors posit that there are two transitions in students' mathematical education. The first transition occurs around the age of 13 to 14 and is commonly referred to as the "transition from arithmetic to algebra". The second transition occurs among older students, aged 17 to 18, and is typically known as the "transition from elementary to advanced mathematical thinking". In this paper, based on relevant literature and practical teaching experience, cognitive conflicts that arise during these transitions will be discussed. Strategies for overcoming these cognitive conflicts will also be proposed.

Key words: *cognitive conflicts, mathematical thinking, phases in mathematical education.*

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The Use of Multiple-Choice Tasks in Mathematics Teaching

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The subject of research in this paper is multiple-choice tasks, whose advantages include: ease of assessment, objectivity in grading, ability to test diverse content, and faster response times. Competitions like "Kangaroo without Borders" exclusively feature multiple-choice tasks, while TIMSS research covers both multiple-choice and open-ended tasks. In Serbia's final exams, the number of multiple-choice tasks is increasing, and these tasks are reviewed automatically. However, in educational practice, open-ended tasks are predominantly used. This study aims to explore the success and behavior of older elementary school students when solving multiple-choice tasks, comparing them with equivalent open-ended tasks. This is examined through the percentage of students with correct results, work methods, and the potential use of different strategies. The analysis shows that students generally achieve better results with multiple-choice tasks compared to open-ended tasks, but they do not apply strategies using the provided answers; rather, random guessing is common. Applying these results to multiple-choice tasks aims to improve teaching practices and learning, enhancing the educational process overall.

Key words: multiple-choice tasks, open-ended tasks, percentage of correct answers, work methods, solving strategies

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Some applications of integral calculus in teaching mathematics in basic studies of economics and business

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One of the challenges in teaching mathematics in basic studies of economics and similar profiles is how to achieve good communication with students who, for various reasons, did not acquire sufficient quality knowledge in mathematics in their previous schooling. Having this in mind, it is necessary to demonstrate to students through teaching how specific mathematical concepts can be applied as tools to solve real-world problems in the field of education, thus directing them towards practical skills. It should be noted that the subject of mathematics is mostly taught in the first semester at the observed faculties, so it is not straightforward to introduce certain mathematical concepts due to time constraints. In this paper, we will present several examples of applying integral calculus in economics and business, which include: determining functions of total accumulated cost, revenue, and profit using corresponding marginal functions; calculating consumer and producer surplus; assessing the time value of capital; and comparing levels of social status. We will use software such as Wolfram Mathematica and Excel for visual interpretation of these examples. Student surveys were conducted in the scope of Business Mathematics course held within the study program of Hospitality and Tourism at the Faculty of Hotel Management and Tourism in Vrnjačka Banja, University of Kragujevac. The results of these surveys on the evaluation of teaching indicate that students have a positive attitude towards the content of the subject and the method of presentation. Finally, the paper will formulate recommendations focusing on innovative approaches in teaching mathematics within the observed study profiles.

Key words: teaching methodology in mathematics, applications of integral calculus, definite integral, economic functions, indicators

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Differences in achievements and trends in state competitions among elementary school students in the Republic of Serbia

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In this study, we will analyze the achievements of students and investigate the existence of trends in national mathematical competitions among primary school students from 2007 to 2024. The population consists of 5534 students from grades 6th to 8th. Quantitative and qualitative research methods were applied. The results indicate that there are differences in student achievements across the observed categories, and trends can be observed in terms of student participation based on gender.

Key words: mathematical competitions, national competition, primary school, student achievements, trend analysis

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Advanced Functions and Software Tools in Financial Mathematics: Impact on Accuracy and Modeling in Education

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This paper explores the application of advanced functions and software tools in financial mathematics and their impact on the modeling process in an educational context. The focus is on integrating new functions within Excel related to loan amortization, bonds, criteria for investment evaluation, etc., into the teaching process of Financial Mathematics courses. Within this study, we conducted a comparative analysis of the accuracy of results obtained by students using advanced functions versus results achieved through traditional learning methods in educational contexts. We also analyzed the influence of those advanced functions on decision-making in financial modeling. The paper provides insights into the effectiveness of these new functions in teaching financial mathematics and their practical application.

Key words: *Software functions in education, Financial mathematics, Modeling*

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Graph Theory in Supplementary Mathematics Classes

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Graph theory is a significant part of supplementary education in primary and secondary schools in the Republic of Serbia. In primary school, this topic usually appears through supplementary classes from the fifth to the eighth grade, while in secondary school, it is covered in the first grade, also as a part of supplementary education. Graphs are of particular importance for working with gifted students because they allow the modelling of various practical problems and the development of generalization in thinking. This presentation introduces some of the important topics from graph theory that are an integral part of supplementary mathematics classes, as well as possible directions for enhancing mathematics education through the application of graph theory.

Key words: *Supplementary mathematics education, graphs*

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High school students' attitudes on the connection between mathematics and chess

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Mathematics and chess have much in common. The way of thinking of mathematicians and chess players are very similar, which is why it is not surprising that mathematicians have had considerable success in chess competitions. Former world chess champions Lasker and Euwe were professional mathematicians, while Tal and Karpov demonstrated exceptional mathematical abilities in their youth. Renowned mathematicians Euler and Gauss worked on combinatorial problems with chess pieces on the board – the former on knight's tours and the latter on the arrangement of queens on the chessboard. By their nature, mathematical problems on the chessboard delve into various mathematical disciplines. Initially, these problems were mostly of a combinatorial, arithmetic, or geometric nature. Later, chess problems began to be approached from the perspectives of graph theory and cybernetics. All this implies a connection between chess and mathematics education. The subject of this research is the attitude of high school students towards the impact of chess on learning mathematics and the need for and manner of introducing chess into the high school curriculum.

Key words: *mathematics education, chess, problem-based learning*

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SECTION 2: SCHOOL PRACTICE - YESTERDAY, TODAY, TOMORROW

Contributions to mathematics teaching: differential and integral calculus

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Differential and integral calculus represent one of the main tools for solving numerous problems in modern science and technology. Real-world problems should serve as a starting point for generalization (or specialization), establishing connections between mathematical disciplines, uncovering mathematical structures, and posing new problems. Practice has shown that the content of differential and integral calculus is often approached superficially and in a formulaic manner. In this paper, special emphasis will be placed on the interconnectedness of concepts, identifying relationships between problems and results, creating new knowledge, and the possibility of their numerous applications, with the aim of highlighting the importance of mathematical reasoning as opposed to the mechanical application of this mathematical apparatus. Through numerous examples and counterexamples, some inconsistencies and inaccuracies in presenting these teaching contents will be pointed out.

Key words: *limit value, function derivative, indefinite and definite integral*

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Variations of STREAM on the topic of the area of squares and rectangles

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In Serbia, STEM, STRAM, or STREAM concepts are rarely used, and even less so in mathematics education, as evidenced by the fact that in the latest catalog of professional development programs for teachers published by ZUOV, only one topic (out of 1216 accredited) is dedicated to STEM competencies. The aim of this communication is to present one of the possible approaches to implementing the teaching topic "Area of squares and rectangles" in the 4th grade of primary school. The model to be presented is based on the STREAM educational concept and includes a general approach to the problem, characteristic problems, working and didactic materials for implementing the topic, results of previous research, perspectives on model development, and concluding considerations.

Key words: *STEM, rectangle area, square area*

Analysis of elementary school students' success in solving discrete mathematics problems in competitions

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The paper will conduct a qualitative and quantitative analysis of elementary school students' success in mathematics competitions when solving problems from various areas of discrete mathematics, such as number theory, logical-combinatorial problems, and the application of graph theory. The analysis will include different types of problems, considering the students' grade level and gender.

Key words: *discrete mathematics, mathematical competitions, elementary school, analysis*

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PARISmatic eTwinning

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Mathematics is not just a tool for calculating, analyzing, comparing, etc., but it is also a subject through which students discover "new worlds", explore new spaces, broaden their horizons, and collaborate with peers across various meridians. At the same time, mathematics allows students professional orientation by placing them in different roles and positions through project activities, enabling them to search for the best solutions. The goal of mathematics education is to equip students to:

- Solve problems and tasks in new and unfamiliar situations;
- Express and explain their opinions and discuss with others;
- Develop motivation for learning and interest in subject content;
- Apply acquired mathematical knowledge in solving various practical life tasks.

The eTwinning project "Discovering Mechanics: Louis Renault" was implemented by the students of Elementary School "Svetislav Golubović Mitraljeta" from Batajnica (Serbia) and the students of Collège Jean Renoir, Boulogne-Billancourt (France). The aim of the project was for students to better understand the principles of mechanics, the cultural and industrial (Renault) heritage of France, and to improve their foreign language skills. Additionally, the idea was to enhance students' mathematical skills. This was precisely my idea when I ventured into eTwinning: to enable my students to collaborate with peers from different parts of Europe through virtual projects, enhancing their language, mathematical, digital, and social skills. This time, the project focused on one of the most significant European cities – Paris, to prepare them for future encounters with it, regardless of their elementary school status. Richard Skemp, who wrote the popular book "The Psychology of Learning Mathematics" (1971), stated: "The problems of learning and teaching are psychological problems, and before we make many improvements in mathematics teaching, we need to know more about how it is learned". There are significant differences in students' success in solving mathematical problems using symbolic, graphical, or verbal representations depending on the complexity level of the tasks.

Key words: *eTwinning, project-based learning, skills, functional knowledge*

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Application of Thales' Theorem - Demonstration Class

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The concept of similarity is inherently too abstract and incomprehensible to a large number of students. The implementation of a demonstration class is an attempt to use an example that can be utilized for visual teaching, guiding students to independently grasp the abstract concept of similarity correctly and acquire lasting knowledge. Through a historical overview of Thales, the creator of significant theorems, and interesting fragments from his life, such as determining the height of the Great Pyramid of Giza, determining the distance of a ship from the seashore, the width of a river, or predicting a solar eclipse, an increased interest in this topic can be achieved among students. A fully prepared content of a demonstration class in which the acquired students' knowledge related to similarity is presented in a real situation of solving a concrete problem, modeled on the methodology Thales applied for determining the width of a river on inaccessible terrain. Specifically, the goal is to determine the length of the basketball court in the schoolyard using Thales' theorem, or rather the similarity of triangles. The "scenario" of the class involves teamwork and the division of tasks among all students. Within the realization of such a well-designed class, some of the well-known theorems in mathematics are connected and applied in a concrete situation: the Converse of the Pythagorean Theorem and Cauchy's Theorem on the Normality of a Line and Plane. The need for greater engagement of teachers in the realization of demonstration classes, like the mentioned example, lies in the fact that our students often cannot translate the acquired knowledge from many areas of mathematics into functional competencies necessary for solving a specific mathematical problem.

Key words: *Thales, similarity, demonstration class*

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A visualization of the limit: Experimental Mathematics

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One of the basic concepts of mathematics that refers to the behaviour of a function near a particular point is the limit value of the function – $\lim_{x \rightarrow x_0} f(x)$. For the first time we encounter limit of a function on the mathematics classes in the high school, and later we use limit in other sciences, usually in physics. The understanding the concept of limit value of the function is by no means simple. There is no universal guideline, but visualization can certainly be a useful tool in overcoming difficulties in this process. In this work, an idea with the visualization through a physical experiment is given. An example of measuring the acceleration of the system using Atwood's machine was presented. The system consists of a pulley of radius R and mass M , and two objects with masses m_1 and m_2 that are connected by an inextensible massless string. The acceleration of the system is measured for the variable mass m_2 and behaviour of the function $a(m_2)$ at infinity ($m_2 \rightarrow \infty$) is examined. The graphical presentation of the results illustrates that the limit value of the function gives the horizontal asymptote $a = g$. The proposed concept would enable the introduction of more effective, visual learning strategies not only in the teaching of mathematics but also in other natural sciences.

Key words: *limit value, acceleration, experiment*

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The number e : an experimental evaluation

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One of the most famous numbers in mathematics is Euler's number e . The number e is the base of the natural logarithm and in literature is often called Neper's number. In the work of distinguished mathematician and physicist Jacob Bernoulli, the number e appears as the limit value of a number sequence that Bernoulli studied dealing with the issue of interest. Although it was primarily used for financial calculations, this remarkable number quickly began to be applied in a wide range of natural phenomena and scientific laws of physics, biology, chemistry. The high schools' students approaching the end of their school education know that $\left(1 + \frac{1}{x}\right)^x$ is equal to the famous number $e \approx 2,718...$ This paper describes an experiment with connected vessels in which indirectly appears number e . For example, if in the described experiment, a vessel with an area of 100 cm² is divided into $N = 100$ smaller vessels with an area of 1 cm², we will theoretically reproduce the number e with an accuracy of 0,5%.

Key words: *number e , limit value, experiment with connected vessels*

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Games in Mathematics Teaching

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In today's educational system, there is an increasing emphasis on interactive learning that encourages active student participation in teaching. Games have the potential to engage students in a unique and interesting way, especially in mathematics classes. In this presentation, I will describe how the introduction of classic board games such as "Murder Mystery", "Snakes and Ladders", and "Battleship" can contribute to a better understanding of mathematical concepts by students. Additionally, I will give special attention to the concept of an Escape Room, which can be implemented in mathematics teaching with the goal of motivating students to work and develop creative thinking through solving mathematical puzzles and problems. This lecture aims to demonstrate how games can become a useful tool for learning mathematics and how they can enhance student engagement in the classroom.

Keywords: *Innovations in Teaching, Board Games, Escape Room*

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Similarity of Triangles in Integrative Teaching

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This paper presents an example of integrative teaching, implemented on May 30, 2024, in the Makerspace of the First Grammar School of Kragujevac, as a demonstration class on the topic "Similar Triangles". First-grade students from the Computing Talents Class organized activities in one of five teams, one of which created a model of a geodesic dome. During this demonstration class, speaking in English to their peers and fellow teachers, they presented the similarity of shapes as inspiration in artworks, fractals, real-world examples, and selected tasks. This activity was showcased on the Twinspace of the eTwinning platform to present an innovative approach to teachers and collaborators in the "Math without Frontiers" eTwinning project.

Key words: Integrative Teaching, Similarity of Triangles, Visual Culture, English Language, Interdisciplinary Connections

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Application of Differential Calculus in High School Physics Teaching

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In the teaching of physics at all levels of education, there is a clear lack of correlation between the teaching content of mathematics and physics. This lack of connection is particularly pronounced in high school physics teaching, where the content related to differential and integral calculus is not integrated with the teaching content of various fields of physics covered during high school education. The two most significant consequences of this issue are: 1) the insufficient preparedness of most high school students for physics and related disciplines at the university level; 2) the insufficient connection of differential and integral calculus with students' direct experiences and natural phenomena. This paper will present an example showing the author's attempt to address this problem in a class of students with special abilities in mathematics at the First Grammar School of Kragujevac.

Key words: differential calculus, interdisciplinary connection, application of derivatives and integrals in physics

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Applying Modular Arithmetic in One Unconventional Problem

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Modular arithmetic is a fundamental concept in mathematics that has numerous applications across various fields, including probability. In this contribution we will explore the use of modular arithmetic to solve one probability problem that is relevant to school mathematics.

Key words: *Modular Arithmetic, Probability*

Puzzle Implementation in Mathematics Education

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This presentation will showcase various opportunities for incorporating puzzles into mathematics education, both in geometric and algebraic contexts. It will demonstrate the use of assembling geometric shapes in instructional units such as Angles on Transversals, Polygons, Polynomial Operations, as well as in various extracurricular activities. Additionally, it will illustrate experiences working with puzzles from some exemplary classes. Through play, puzzles introduce children to new mathematics areas or systematize previously covered school material. They also encourage peer learning and teamwork. Both ready-made forms and those crafted by students themselves are utilized, alongside some digital tools (Tarsia). They are suitable for working with children following individual educational plans. The poster will display the materials themselves along with some photographs from these classes.

Key words: *puzzle, geometry, algebra*

Singapore math approach to solving word problems

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A big problem in today's education is that fewer and fewer students understand the content of the text they are reading. This greatly affects the success of solving word problems.

The essence of the principle of Singapore math is to teach students how to turn abstract problems into concrete images that are easily translated into mathematical procedures when solving word problems. Singapore math uses visual models because they are much clearer for students. It is an important step for children between reading word problems and moving on to the steps needed to solve the problem. Through an organized system of steps, students are guided to the solution of the problem by constantly returning to the text of the task, reading several times, and turning abstract concepts into a visual model. By applying Singapore math, all methodical principles crucial for the development of mathematical skills are satisfied: educational orientation, moderation and sequence, obviousness and sensibility, conscious activity, scientificity and systematicity, as well as durability of knowledge. Such work with the youngest students provides a better foundation for further work on mathematics education in later schooling. Singapore math is particularly suitable for practice, determination, and systematization lessons.

Key words: *Singapore math, visualization*

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Fibonacci Sequence, Golden Ratio, and Golden Spiral - Examples

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„Mathematical Fascination“ is a project by the teachers and students of the Technical School in Požega on the topic of the Fibonacci sequence, the golden ratio, and the golden spiral. It was implemented through project-based learning using the STEM concept. Over the course of eight weeks, second-year students at the Technical School in Požega worked on a project that combined mathematics and biology and focused on the golden ratio in geometry and architecture. The students were divided into six groups and carried out research and product development according to given instructions/research tasks: 1. Art and Architecture – They analyzed works that use the golden ratio for aesthetic harmony, as well as architectural examples throughout history; 2. Anatomy – They found examples of the golden ratio in the human body, such as facial proportions or the arrangement of fingers; 3. Nature – They discovered examples of the golden ratio and the Fibonacci sequence in the formations of natural objects, such as hurricanes or galaxies; 4. Plants – They identified Fibonacci sequences in the arrangement of leaves, flowers, or seeds; 5. Algebra – They found the algebraic solution to the golden ratio problem (dividing a segment of length a into parts x and $a-x$ such that $a:x = x:(a-x)$); 6. Geometry – They performed a geometric construction of dividing a segment by the golden ratio (dividing the segment by the golden ratio using a compass and a ruler and proving the correctness of the construction using the similarity of triangles). The students systematized and presented their research results in the form of posters (first, second, third, and fourth groups) and videos (fifth and sixth groups). All the project outputs, as well as the method of outcome assessment, are presented in an e-book:

<https://read.bookcreator.com/reM4j2B1GLfwDXmUZMRALyc5RIp2/AEZc-vfqSqKb2mp9c66WHA?authuser=0>. The achievement of outcomes was verified by organizing an Escape Classroom:

<https://view.genially.com/663de70191e5290014b293f7/interactive-image-escape-classroom-za-matematichka-fascinacia>.

Key words: *golden ratio, Fibonacci sequence, anatomy, architecture, morphology*

Through Mathematical Calculations and Informatic Visualizations to the Microworld of Molecules

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This model class represents an innovative and interactive experiment exploring the reaction of salt with ethanol and aluminum foil, emphasizing the role of mathematics and computer science in data analysis and visualization. The aim of this study was to provide students with the opportunity to develop and apply mathematics and informatics skills through practical work and direct observation of chemical reactions.

The purpose of this work was to familiarize students with methods of quantitative analysis and data visualization, develop critical thinking and problem-solving skills, and integrate mathematics and informatics abilities in a chemical context. This study attempts to answer the following questions: How does sodium chloride react in an ethanol environment in the presence of aluminum foil? How can the results of this reaction be mathematically analyzed and visually represented using computer tools?

Students will use mathematical tools to calculate concentrations, yield percentages, and stoichiometric ratios, enabling precise quantitative analysis of the obtained data. Computer tools such as Microsoft Excel and Python with the Matplotlib and Pandas libraries will be used for tabular analysis, data visualization, and reaction simulation.

The results of this experiment show that specific reactions occur in the presence of aluminum foil, producing new substances. Students will use Excel for systematic recording and analysis of data, while Python will enable the creation of advanced graphical representations and reaction models. These tools will allow them to create detailed reports and visual presentations of the results. In conclusion, not only does this experiment contribute to the understanding of chemical reactions but it also develops key mathematics and informatics skills. This approach to learning integrates chemistry, mathematics, and computer science, making science more accessible and interesting for students. Additionally, it encourages students to think in a multidisciplinary manner and prepares them for complex problems in scientific and technological disciplines.

Key words: *Chemical Reactions, Ethanol, Salts, Microsoft Excel, Python*

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Lego and MASTERPIECE - Digital recording of Serbian tradition

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Aware of the existing quality and continuous desire for the success of our students, for ten seasons, the Lego section of the First grammar school aims to develop creativity, teamwork, and problem-solving skills among students. According to the theme of this year's season, MASTERPIECE, team Lego Musketari worked on a project to create a platform for learning the basic steps of the national dance, Moravac kolo. Considering that the Serbian folk dance – kolo was included on the UNESCO Intangible Cultural Heritage list in 2017, we wanted to encourage the preservation of Serbian tradition and culture through this project. Team Musketari conceived, designed, and programmed the first interactive dancing pad, that helps people to learn basic steps of one of the oldest Serbian dances, combining music and light fields. In this way, through play, students acquired basic knowledge about our cultural heritage, as well as about programming Arduino Uno devices, connecting RGB diodes, relays, with power supply, and synchronizing light fields and music.

The goal of this year's season is to, by creating a digital recording of our traditional dance, bring our dance – kolo closer to all people, giving them the opportunity to delve deeper into our culture.

Key words: STEM, extracurricular activities, peer learning, critical thinking, problem-Solving

Challenges and opportunities: presentation of effective methods of popularizing mathematics through international cooperation and examples from practice

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Teaching mathematics is a challenge in itself. Today, mathematics is not "popular", so it is particularly demanding to engage students with it and motivate them to work and think. Guided by these problems, we came up with the idea of the international eTwinning project MathMay, whose main goal is the popularization and application of mathematics. In this paper some of the project tasks, as well as the method of project implementation itself will be presented. Senior students (aged 11-15) from two elementary schools from Serbia and one from Spain participated in the project. As the nature of the project was multi-disciplinary, the students developed different competencies (digital, maths and science, personal, social etc.), enjoyed and had fun while applying mathematics in the implementation of the project. Therefore, all project tasks were very successfully implemented, and the goals achieved.

Key words: popularization and application of mathematics, eTwinning project, elementary school students.

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Equivalence of propositional functions

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When expressing mathematical content, logical connectives should have a precise logical function. In other words, the meaning of a complex sentence should be unambiguously determined by the meaning of the sentences it is composed of. Such precision must be present in the expression of teachers and should be developed in students as well. The main subject of our work is the precise and meaningful use of logical connectives.

Key words: logical connectives, propositional function, irrational equations, irrational inequalities

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A Platform for Nonlinear Programmed Learning of Mathematics

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This paper presents a platform for nonlinear programmed learning, an innovative tool for individualized acquisition and practice of educational content. The platform allows students to learn and practice various procedures, especially in mathematics education, but also in other subjects, adapting to their needs and levels of knowledge. Nonlinear programmed learning involves generating subsequent tasks based on the student's current response. When solving tasks, students have access to textual or pictorial questions with four possible answers, out of which only one is correct. In the case of a correct answer, the student moves on to a new task. Otherwise, they receive a simpler task or instructions explaining common mistakes, as well as review materials in the form of definitions and statements. The platform enables students to practice independently, both in school and at home, at their own pace, without direct teacher involvement. On the other hand, the teacher's interface allows the input and customization of tasks, the definition of task difficulty levels, and the monitoring of student progress. Teachers have insight into students' results, which allows them to better understand each student's knowledge level and specific needs, through the recorded unique exercise path.

The nonlinear programmed learning platform provides a dynamic and interactive environment that encourages students to engage in active learning and continuous improvement, while providing teachers with tools for effective monitoring and support in the educational process.

Key words: nonlinear programmed learning, interactive platform, modern tools

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STEM Approach as a Means of Motivation in Teaching Mathematics

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One of the biggest challenges faced by today's mathematics teachers is motivating students and giving meaning to their learning. This problem is even greater if a student initially perceives mathematics as a difficult science to master. As an adequate response to existing problems, the STEM (Science, Technology, Engineering, Mathematics) approach to teaching imposes itself with its general characteristics. By placing the student at the center, this approach significantly contributes to improving the quality of the teaching process itself, thereby raising students' competencies and achievements to a higher level. The student is the one who explores, experiments, solves problems, draws conclusions and makes decisions, and sometimes makes mistakes and learns from those mistakes. Through its interdisciplinarity, STEM provides students with an answer to the famous question: "Why am I learning this?" and develops the appropriate STEM and mathematical skills necessary for solving real-life and abstract problems. On the other hand, the STEM approach also requires appropriate teacher competencies and their continuous professional development through various training sessions and exchanges of experiences with other colleagues. This paper presents the STEM approach in teaching mathematics in primary school through an example of good practice and analyzes how and to what extent its application contributes to acquiring lasting and functional knowledge for students. It also discusses the possibility of implementing the STEM approach in the regular curriculum and its long-term positive impact on developing skills necessary for the 21st century.

Key words: *STEM, mathematics, teaching, didactics, methodology*

Mathematics teaching in the Women's High School (1863-1904)

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Almost nothing is known about the work of the first educational institutions (schools) intended for the education of girls in the 19th century, even today, especially from a didactic-methodical point of view. In order to better understand the journey from the creation of the first to the most modern schools (Đorđević, Lukić, 2018: 79), it is important to understand the curricula, programs and teaching aids that were applied at that time. By reviewing the relevant scientific literature, one can come across rare data that indicate the way of organizing teaching, but without any deeper analysis of teaching plans, programs and resources. This state of affairs can be understood to some extent, because at that time, girls acquired the right to education much later than boys. The fact that the first girls' school was opened long before the law on the opening of girls' primary schools was passed speaks volumes about the position of girls (Vasojević, 2014). It was only in the sixties of the 19th century that the higher education of female youth began to be considered, and in 1863 Prince Mihailo signed the Law on the opening of the first Women's High School in Belgrade (Knežević-Lukić & Vasojević, 2021). The paper will show the representation of mathematics teaching in the Women's High School, with a special focus on the teaching content that was applied in practice. In this way, a special theoretical contribution will be made to the history of pedagogy and mathematics teaching methodology.

Key words: *Women's High School, mathematics teaching, girls' education*

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ESCAPE ROOM

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In Serbia, students have a low level of digital literacy. For example, they do not know how to react when inappropriate content appears on the screen, they do not know how to deal with large amounts of information on the Internet, how to validate it and turn it into knowledge. That is why it was very important that I create together with my students a virtual educational package – "Escape room", in which we transform teaching topics into a form that is understandable and receptive to today's generation. Given that the Minecraft environment is the most used in the educational world today, it was used to transform and adapt the mandatory teaching material on Digital Literacy into digital content in the form of virtual rooms. Each room represents one topic of the teaching material, and each room is designed to look like the topic the students will be learning about. The virtual world "Escape room" consists of seven rooms that cover seven topics related to digital literacy, which is a part of the curriculum of the subject Informatics and Computing for elementary school. Each room presents scenarios that mimic real Internet risks. For example, "Online Identity" simulates a catwalk, illustrating potentially dangerous situations on the Internet when security measures are not followed, offering students and teachers an exciting learning experience.

In each room, learning content is presented by "experts" along with questions related to the topic. To progress, students must answer all questions to move to the next room. Additionally, each room includes a visual self-assessment of the student's knowledge. At the exit of the room, the student chooses a brick that characterizes their achievement.

Key words: *digital literacy, open educational package, Minecraft*

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A Decade of STEAM Development Through Extracurricular Activities - Lego

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During the ten-year duration of the First Lego League (FLL) extracurricular activities, the focus was on improving STEAM education among students. The main goal of this extracurricular activity was to encourage and foster creativity, teamwork and problem-solving skills in students through challenges based on the application of Lego robotics. We organized workshops and participated in various presentations and national, regional and world competitions. During all these activities, significant progress was observed in the development of skills among the students, as well as an increased interest in STEAM fields. Additionally, through peer education, older students mentored younger ones, sharing their knowledge and experience, which further strengthened team spirit and supported the continuous development of skills, from critical thinking, teamwork to technological literacy among children. The implementation of FLL practice involves the integration of innovative methods in the field of extracurricular activities in order to support the development of student competencies needed for future challenges. Through the activities of the extracurricular activities, significant results have been achieved in stimulating interest in STEAM fields among students, indicating the importance of continuing such initiatives in schools. These activities have contributed to creating a stimulating environment for learning and developing critical thinking in children, ensuring better preparation for future academic and professional challenges.

Key words: STEAM, extracurricular activities, peer learning, critical thinking, problem-solving

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Royal game of ur and mathematics

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Most educators consider motivation as a key precondition for successful learning. Nowadays, especially in older age groups, it is very difficult to engage students in learning mathematical content. This paper will describe the Royal Game of Ur, the oldest known board game originating from ancient Mesopotamia. We will also present several problems from probability theory inspired by this game. This could be a subject of research on the impact of this or similar games on motivation for learning mathematics. Finally, an electronic version of the Royal Game of Ur, modeled by the author in the GeoGebra software package, will be introduced.

Key words: *Royal Game of Ur, probability, GeoGebra, motivation*

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